Wave Analysis of Oscillatory Fingers in Microchannel

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Abstract: We present an experimental study and characterize the dynamics of the propagation modes that occur due to the displacement of a viscous fluid by an air finger through microchannels with centered rectangular occlusions. Furthermore, we extent the result of our recent work [1] by investigating the wave front exhibited by oscillatory fingers as the capillary number $Ca = \frac{\mu U}{\sigma}$ (which is the ratio of viscous to surface tension forces, where $\mu$ is the fluid viscosity, $U$ is the velocity of the finger and $\sigma$, the surface tension) increases. We found that the wavelength of an oscillatory finger decreases as the $Ca$ increases while $L_{tip}$ increases with $Ca$.

Keywords: Bifurcation, Fingers, Microfluidics, Stability, Transition.

1. Introduction

Efficient knowledge of the behavior of flow of confined bubbles and droplets within natural and man-made channels is essential for the design of microchannel and lab-on-a-chip applications [2]. Experimental investigation of bubble flow in microchannels reveal that bubbles exhibit different morphological states such as symmetric, asymmetric, localized and oscillatory [1-3]. Similar study by [4] revealed that for sufficiently high occlusions, two alternative stable solutions are possible for low flow rates: a steadily propagating asymmetric air finger that spans the majority of the tube or a steadily propagating asymmetric air finger that localized within one of the side-channels. In this study, we present further results of our recent work [1] which focused on the analyses of wave fronts in oscillatory bubbles. Specific applications of oscillatory bubbles include pumps, mixers, filters and transporters [5].

2. Experimental Set-up

The schematic channel diagram of the experimental setup is shown in figure 1.
Figure 1 (a) illustrates how liquid is withdrawn at constant flow rate \( Q \) from one end of the initially liquid-filled tube, with two of the outlets at the other end of the channel occluded for the production of an air finger. Figure 1 (b) is the cross section of the channel. The outer width and height of the cross section are \( W \) and \( H \) respectively, while the corresponding centered rectangular occluded cross section are \( w \) and \( h \) respectively.

The channel was manufactured by micro-milling of a piece of perspex (CAT3D-M6, CNC, milling machine, Datron Technologies Ltd), and sealed with a clear adhesive film (Corning), which was supported by a precision-milled flat perspex lid. There are three channels labelled Channels 1, 2 and 3. The channels have constant height and length of 300 ±5 \( \mu \)m and 9.60 ±0.05 cm respectively with a rectangular centered partial occlusion each having constant length of 9.0 ±0.05 cm thereby leaving a length of 0.60 ±0.05 cm un-occluded space in all channel to allow initial states of the fingers to be symmetric before driving them through the occluded part of the channels. The widths of both the channels and the obstacles were measured with a travelling microscope. Refer to [1] for more details and mechanism of the experimental set-up.

3. Results and Discussion

3.1. Multiple Family of Propagating Fingers

In this section we present Top-views of the different types of fingers observed in our experiments. The nature of these fingers and their oil recovery properties is a function of a bulk measure of the fluid displacement given by the wet fraction.

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(m = 1 - Q/AU, \text{ where } Q \text{ is the flow rate, } U \text{ is the velocity of the finger and } A = WH - wh \text{ is the cross-sectional area of the tube})
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and are shown in figures (2 - 7). The arrows indicate the direction of motion of the finger.

**Figure 2**: Symmetric finger; the arrow indicates the direction of flow in all cases. \( Ca = 0.0026 \) Channel 2

**Figure 3**: Asymmetric finger. \( Ca = 0.014 \) Channel 1

**Figure 4**: Localized finger. \( Ca = 0.032 \) Channel 3

**Figure 5**: Oscillatory finger. \( Ca = 0.057 \) Channel 2

3.2. Oscillatory Fingers

Further experiments and measurements were carried out to investigate the oscillatory fingers observed in channel 2. The oscillatory propagation mode is the most striking and complex feature observed in our experiments. In this context, we define ‘oscillatory’ to mean when the moving finger tip leaves behind a spatially varying periodic disturbance which then remains stationary in the laboratory frame. The disturbance at the interface occurs along one side of the finger, where the wave amplitudes seen are centered along the edge of the obstacle. The other side of the finger lies close to the channel wall. Consequently, the finger tip spans the region over the obstacle and one of the two un-occluded regions.
Oscillations emerged at high capillary numbers i.e. Ca>0.032. As Ca increases, there is a significant reduction in the variation of the tip shape over the period of oscillations. This mechanism for the onset of oscillations is consistent with that proposed by [6] in millimetric channels. For large-amplitude oscillations, our finger interface shapes are similar to a square wave due to the channel walls as shown in fig. 5. We noted that for air fingers, oscillations are always initiated near the finger tip and propagates backwards [7]. We investigate the wavelength of the finger for different flow rates as well as the finger tip length (Ltip) (define as the distance from the tip position of the finger to the point beyond which oscillation occurs) with MATLAB inbuilt ‘imdistline’ function. We determine the wavelength by measuring the length of three consecutive fully developed wave lengths behind the Ltip and averaged it. This was done for the different images recorded for a range of flow rates with constant increment of Q=30μl/min. From our experimental results presented in figure 6 shows that as the capillary number exceeds the critical capillary value of Cac = 0.032, the axial symmetry of the finger is broken, which almost smoothly transcends to asymmetric non-localized finger in which the tip propagates with an almost constant shape and speed, but leaves behind a spatially varying periodic disturbance. This unique but complex finger propagation mode is described by [8] to have manifested due to the existence of a symmetry-breaking pitchfork bifurcation between the symmetric and asymmetric steady states while [6] suggested that a global homoclinic connection gave rise to the oscillatory propagation modes. In figure 7, as the capillary number increases, there is rapid oil evacuation from the un-occluded region once the interface has passed over the edge of the obstacle, thereby, causing an increase in the frequency of oscillations and subsequent decrease in wavelength. This is why the wavelength decreases as Ca increases and may even cease to exist for high enough flow rates, as reported by [6] in millimetric channels. Conversely, Ltip has direct proportionality with Ca. The increase in the capillary number causes the finger to broaden more slowly across the obstacle, which induces a more rapid change in curvature; and as a result, Ltip increases with increasing Ca. See figure 8.

Figure 6: Channel 2: Wet fraction m as a function of capillary number Ca. The insets are top-view images of air fingers propagating through the channel from right to left. Each data point represents a single experiment conducted at the end of the channel.
Figure 7: Characteristic wavelength (mm) obtained by averaging the length of three consecutive fully developed wavelengths behind the Ltip, plotted as a function of Ca.

![Characteristic wavelength graph](image)

Figure 8: Characteristic tip length (Ltip) (the distance from the tip to the position beyond which oscillations occur), plotted as a function of difference between the capillary number and the critical capillary number Cac. Each data point represents a single experiment.

![Characteristic tip length graph](image)

4. Conclusion

The results of wave analysis of oscillatory bubbles in micro-channel have been presented. A simple modification of the tube geometry is evident to have fundamentally altered the dynamics of bubble propagation induced by constant flux. In addition to the single, symmetric mode seen in un-occluded channels, there exists asymmetric, localized and oscillatory modes in our occluded microchannels analogous to the family of propagating fingers recently uncovered by [6, 9] in millimetre-scale tubes, indicating that gravity is not an essential physical mechanism that underpins the emergence of these states. We observed symmetric finger at low capillary numbers which lost symmetry to either asymmetric, localized or oscillatory finger through supercritical symmetry-breaking bifurcation as the capillary number increases beyond a given threshold. Our results revealed that symmetric fingers are noted for greater oil recovery property while the asymmetric, localized and oscillatory fingers limit the amount of liquid recovered. These bubble propagation modes offer further potential for geometry-induced manipulation of droplets for lab-on-chip applications. Furthermore, our results have shown that the wavelength of an oscillatory finger decreases as the Ca increases while Ltip increases with Ca.
References


